第二届最优传输、随机矩阵及相关问题研讨会

中国科学院数学与系统科学研究院

2023.12.8-10

➢ 报告人

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➢ 组委会

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➢ 基金支持

- 国家重点研发项目：随机分析的基础理论研究
- 中科院数学与系统科学研究院卓越团队项目
- 中科院数学与系统科学研究院随机分析研究中心

➢ 报告日程

时间\会期	12月8日南楼202	12月9日南楼219	12月10日南楼219
08:30 - 09:05	王凤雨	焦　勇	朱蓉禅
09:05 - 09:40	刘　伟	王才士	苏中根
10:00 - 10:35	华波波	王振富	张　登
10:35 - 11:10	王　东	刘党政	杨　帆
11:25 - 12:00	吴　波	王宇钊	吕　琦
14:30 - 15:05	胡泽春	王　健	
15:05 - 15:40	李　娟	丁　昊	
16:00 - 16:35	陈世炳	马宇韬	
16:35 - 17:10	郑术蓉	张朝恩	
17:25 - 18:00	张希承	韩邦先	

➢ 报告日程

	08:20 - 08:30		
	08:30 - 09:05	王凤雨	Diffusion processes on Wasserstein space
	09:05 - 09:40	刘 伟	Smoluchowski-Kramers approximation for distribution dependent stochastic differential equations
	09:40 - 10:00		茶歇
	10:00 - 10:35	华波波	Liouville theorems for graphs with nonnegative curvature
	10:35 - 11:10	王 东	Local hard edge universality of Muttalib-Borodin ensemble
	11:10 - 11:25		茶歇
12月8日 南楼202	11:25 - 12:00	吴 波	Weak Lévy-Gromov's isoperimetric inequality
	14:30 - 15:05	胡泽春	Favorite edges and rarely visited edges of the simple random walk
	15:05 - 15:40	李 娟	Mean field stochastic control problems under sublinear expectation
	15:40 - 16:00		茶歇
	16:00 - 16:35	陈世炳	Regularity of singular set in optimal transport
	16:35 - 17:10	郑术蓉	Large separable sample covariance matrices: joint CLT for linear spectral statistics and its applications
	17:10 - 17:25		茶歇
	17:25 - 18:00	张希承	SDEs with supercritical distributional drifts
	08:30 - 09:05	焦 勇	The sharp weighted maximal inequalities for noncommutative martingales
	09:05 - 09:40	王才士	Anderson model and infinite dimensional hypercube
	09:40 - 10:00		
	10:00 - 10:35	王振富	Quantitative propagation of chaos for 2D viscous vortex model on the whole space
	10:35 - 11:10	刘党政	From Wigner matrices to Anderson models
	11:10 - 11:25		茶歇
12月9日 南楼219	11:25 - 12:00	王宇钊	Entropy formula and displacement convexity for p-Laplacian on Wasserstein spaces over Riemannian manifolds
	14:30 - 15:05	王 健	Quenched local limit theorem for random conductance models with long-range jumps
	15:05 - 15:40	丁 昊	A particle approximation to the Dean-Kawasaki equation
	15:40 - 16:00		茶歇
	16:00 - 16:35	马宇韬	Importance sampling estimator for rare events of β-Jacobi ensembles
	16:35 - 17:10	张朝恩	Towards quantitative H theorems: hypocoercivity and functional inequality
	17:10 - 17:25		茶歇
	17:25 - 18:00	韩邦先	Barycenter and optimal transport
	08:30 - 09:05	朱蓉禅	Large N limit and $1/N$ expansion of the observables for $O(N)$ linear sigma model
	09:05 - 09:40	苏中根	How big are the increments of Airy process?
	09:40 - 10:00		茶歇
12月10日 南楼219	10:00 - 10:35	张 登	The three dimensional stochastic Zakharov system
	10:35 - 11:10	杨 帆	Bulk universality and quantum unique ergodicity of random band matrices
	11:10 - 11:25		茶歇
	11:25 - 12:00	吕 琦	Exact controllability for a refined stochastic wave equation

➢ 报告题目与摘要

陈世炳（中国科学技术大学）

Title: Regularity of singular set in optimal transport

Abstract: In this talk, I will discuss a regularity theory for the optimal transport problem when the target consists of two disjoint convex domains, denoted Ω_i^* for $i = 1,2$. This is a fundamental model in which singularities arise. The regularity of the singular set is derived from the regularity of the potential functions up to it. We introduce a novel framework to demonstrate the higher regularity of the potential functions up to the singular set, despite the singular set exhibiting no form of convexity a priori, including semiconvexity. Notably, our results are achieved without requiring any convexity of the source domain. This aligns with Caffarelli's celebrated regularity theory.

丁　昊（中科院数学院交叉中心）

Title: A particle approximation to the Dean-Kawasaki equation

Abstract: Following the idea of stochastic parallel translations on Wasserstein space, we construct a new particle approximation to the regularized martingale solution of the diffusive Dean-Kawasaki equation on 1-D torus with colored noise.

韩邦先（山东大学）

Title: Barycenter and optimal transport

Abstract: Barycenter is a basic concept in both geometry and probability theory. For example, as the solution of a simple optimization problem, barycenters provide a natural generalization of the notion of mean value of a probability measure, as well as the expectation of metric-space valued random variables. In this talk, I will introduce some new results on its relationship with optimal transport.

➢ 报告题目与摘要

胡泽春（四川大学）

Title: Favorite edges and rarely visited edges of the simple random walk

Abstract: In this talk, we will introduce our recent results on favorite edges and rarely visited edges (i.e., edges that are visited only once) of the one-dimensional simple symmetric random walk based on the following two papers:

[1] C.-X. Hao, Z.-C. Hu, T. Ma, R. Song: Three favorite edges occurs infinitely often for one-dimensional simple random walk, Accepted by Communications in Mathematics and Statistics, 2023.

[2] Z.-C. Hu, X. Peng, R. Song, Y. Tan: The asymptotic behavior of rarely visited edges of the simple random walk, arXiv: 2310.16657v1 (2023).

In [1], we showed that with probability 1 three favorite edges occurs infinitely often. Let $\alpha(n)$ be the number of rarely visited edges up to time n. In [2], first we showed that $n \to \mathbb{E}(\alpha(n))$ is non-decreasing in n and that $\lim_{n \to +\infty} \mathbb{E}(\alpha(n)) = 2$; then we studied the asymptotic behavior of $\mathbb{P}\left(\alpha(n) > a(\log n)^2\right)$ for any $a > 0$ and used it to show that there exists a constant $C \in (0, +\infty)$ such that $\lim_{n \to +\infty} \frac{\alpha(n)}{(\log n)^2} = C$ almost surely.

华波波（复旦大学）

Title: Liouville theorems for graphs with nonnegative curvature

Abstract: For a graph with nonnegative Bakry-Emery curvature or Ollivier curvature, we discuss some known results on Liouville theorems for discrete harmonic functions. Moreover, we prove that there are at most two ends of infinite volume for the Ollivier case via the existence of nontrivial linear growth harmonic functions. The talk is based on joint works with Florentin Muench.

焦　勇（中南大学）

Title: The sharp weighted maximal inequalities for noncommutative martingales

Abstract: The purpose of this talk is to present weighted maximal L_p-inequalities in the context of operator-valued martingales on semifinite von Neumann algebras. The main emphasis is put on the optimal dependence of the L_p constants on the characteristic of the weight involved. As applications, we establish weighted estimates for the noncommutative version of Hardy-Littlewood maximal operator and weighted bounds for noncommutative maximal truncations of a wide class of singular integrals.

李 娟 (山东大学)

Title: Mean field stochastic control problems under sublinear expectation

Abstract: In this talk we study Pontryagin's stochastic maximum principle for a mean-field optimal control problem under Peng's G-expectation. The dynamics of the controlled state process is given by a stochastic differential equation driven by a G-Brownian motion, whose coefficients depend not only on the control, the controlled state process but also on its law under the G-expectation.

Also the associated cost functional is of mean-field type. Under the assumption of a convex control state space we study the stochastic maximum principle, which gives a necessary optimality condition for control processes. Under additional convexity assumptions on the Hamiltonian it is shown that this necessary condition is also a sufficient one. The main difficulty which we have to overcome in our work consists in the differentiation of the G-expectation of parameterized random variables.

Based on a joint work with Rainer Buckdahn (UBO, France), Bowen He (SDU, China).

刘党政 (中国科学技术大学)

Title: From Wigner matrices to Anderson models

Abstract: In the 1950s Wigner and Anderson independently brought randomness into Hamiltonians in quantum physics, which give birth to Wigner matrices and Anderson models. Both seem quite different: more randomness or less randomness, dense structure or sparse structure, mean-field or not. However, the ties between the two may run deep.

刘 伟 (武汉大学)

Title: Smoluchowski-Kramers approximation for distribution dependent stochastic differential equations

Abstract: In this talk, we will show the convergence rates in the Smoluchowski-Kramers approximation for the solution of the distribution dependent SDE driven by Brownian motion or fractional Brownian motion.

吕 琦 (四川大学)

Title: Exact controllability for a refined stochastic wave equation

Abstract: A widely used stochastic wave equation is the classical wave equation perturbed by a term of Itô's integral. We show that this equation is not exactly controllable even if the controls are effective everywhere in both the drift and the diffusion terms and also on the boundary. In some sense this means that some key feature has been ignored in this model. Then, based on a stochastic Newton's law, we propose a refined stochastic wave equation. By means of a new global Carleman estimate, we establish the exact controllability of our stochastic wave equation with three controls. Moreover, we give a result about the lack of exact controllability, which shows that the action of three controls is necessary. Our analysis indicates that, at least from the point of view of control theory, the new stochastic wave equation introduced in this paper is a more reasonable model than that in the existing literatures.

马宇韬 (北京师范大学)

Title: Importance sampling estimator for rare events of β-Jacobi ensembles

Abstract: We construct the importance sampling estimator (IS in short) of rare events relating to the top eigenvalues of β-Jacobi ensembles. For general case, we establish a new large deviation principle for the top eigenvalues and then we obtain the logarithmical efficiency of the IS. Meanwhile, for the very rectangular case, we show the strong efficiency of the IS and also prove that the IS converges to the optimal estimator with respect to the total variation distance.

苏中根 (浙江大学)

Title: How big are the increments of Airy process?

Abstract: The Airy process is a real valued random process whose finite dimensional distribution is determined by a Fredholm determinant with Airy kernel. It was first introduced by Prahöfer and Spohn in the study of polynuclear growth model more than 20 years ago and has become a central object in the KPZ universality class. There has been some intensive research activities around the Airy process, some of which has rigorously proved its existence, time correlation and continuity, and more interestingly obtained the modulus of continuity. Compared to well-studied Brownian motions, Brownian bridges and even Ornstein-Ulenbeck processes, Airy process and its extension (i.e. Airy line ensembles) are new, so it is worthwhile further research. In this talk I shall briefly review some remarkable results in this field with focus on the increments of Airy sample paths, no detailed proofs are given.

王才士 (西北师范大学)

Title: Anderson model and infinite dimensional hypercube

Abstract: The classical Anderson model is actually a random Schrödinger operator on the d-dimensional integer lattice Z^d, which plays an important role in the physics of condensed matter and disordered systems. In this talk, we first introduce an infinite graph, which we call the infinite-dimensional hypercube (IDH). Then we introduce an IDH-version of the classical Anderson model, namely a random Schrödinger operator on the IDH. Among others, we show the ergodicity of the model and the existence of its density of states measure.

王 东 (中国科学院大学)

Title: Local hard edge universality of Muttalib-Borodin ensemble

Abstract: Muttalib-Borodin ensemble is defined by the joint probability density function

$$\prod_{1 \le i < j \le n} (x_i - x_j)(x_i^\theta - x_j^\theta) \prod_{i=1}^{n} e^{-nV(x_i)}.$$

It is proposed by physicist Muttalib as a toy model of quantum transport, and has relations to random matrix theory. Because of its simplicity and its non-trivial hard edge limit, Muttalib-Borodin ensemble becomes the archetype of biorthogonal ensembles. Borodin studied this model in the $V(x) = x$ case, and found its limiting distribution around the hard edge 0. We show that for a large class of V, Muttalib-Borodin ensemble has the same limiting distribution, that is, the model has a universal property. Our approach is by the asymptomatic analysis of a kind of vector-valued Riemann-Hilbert problem.

王凤雨 (天津大学)

Title: Diffusion processes on Wasserstein space

Abstract: By using the intrinsic derivative and reference probability measures on the Wasserstein space transformed from the tangent space, we construct a class of conservative quasi-regular local Dirichlet forms on the Wasserstein space. In particular, the O-U type Dirichlet form on the Wasserstein space satisfies the log-Sobolev inequality, the associated O-U semigroup is compact, and the heat kernel has a nice L^2-upper bound.

王 健 (福建师范大学)

Title: Quenched local limit theorem for random conductance models with long-range jumps

Abstract: We establish the quenched local limit theorem for random walk among stationary ergodic random conductances that permit jumps of arbitrary length. The proof is based on the weak parabolic Harnack inequalities and on-diagonal heat-kernel estimates for long-range random walks on general ergodic environments.

王宇钊 (山西大学)

Title: Entropy formula and displacement convexity for p-Laplacian on Wasserstein spaces over Riemannian manifolds

Abstract: In this talk, we prove the W-entropy monotonicity formula for p-Laplacian along the geodesic flow on the L^q-Wasserstein space over n-dimensional compact Riemannian manifolds, where $1/p + 1/q = 1$. Moreover, we study the generalized displacement convexity for nonlinear mobility continuity equation with p-Laplacian on Wasserstein space over Riemannian manifolds.

王振富 (北京大学北京国际数学研究中心)

Title: Quantitative propagation of chaos for 2D viscous vortex model on the whole space

Abstract: We derive the quantitative estimates of propagation of chaos for the large interacting particle systems in terms of the relative entropy between the joint law of the particles and the tensorized law of the mean field PDE. We resolve this problem for the first time for the viscous vortex model that approximating 2D Navier-Stokes equation in the vorticity formulation on the whole space. We obtain as key tools the Li-Yau-type estimates and Hamilton-type heat kernel estimates for 2D Navier-Stokes on the whole space. This is a joint work with Xuanrui Feng from Peking University.

吴 波 (复旦大学)

Title: Weak Lévy-Gromov's isoperimetric inequality

Abstract: In this talk, we will introduce a weak Lévy-Gromov's isoperimetric inequality for the diffusion operator. Moreover, we also present some connections between weak Lévy-Gromov's isoperimetric inequalities and other functional inequalities.

杨 帆 (清华大学)

Title: Bulk universality and quantum unique ergodicity of random band matrices

Abstract: Consider a general class of random band matrices H on the d-dimensional lattice of linear size L. The entries of H are independent centered complex Gaussian random variables with variances s_{xy}, which have a banded profile so that s_{xy} is negligible if $|x - y|$ exceeds the band width W. In dimensions $d \geq 7$, assuming that $W \geqslant L^\delta$ for a small constant $\delta > 0$, we prove the delocalization and quantum unique ergodicity (QUE) of the bulk eigenvectors of H. Furthermore, we prove the bulk universality of H under the condition $W \gg L^{95/(d+95)}$. In the talk, I will discuss a new idea for the proof of the bulk universality through QUE, which verifies the conjectured connection between QUE and bulk universality. The proof of QUE is based on a local law for the Green's function of H and a high-order T-expansion developed recently. Based on Joint work with Xu, Yau and Yin.

张朝恩 (哈尔滨工业大学)

Title: Towards quantitative H theorems: hypocoercivity and functional inequality

Abstract: Boltzmann's H theorem asserts that the entropy is monotone along the evolution of the Boltzmann equation. It is interesting to obtain quantitative versions of the H theorem for kinetic models, in other words, to obtain explicit convergence to equilibrium in the sense of entropy. One of the most famous results in this direction is the logarithmic Sobolev inequality for the Gaussian distribution, which implies exponential decay in entropy for the Ornstein-Uhlenbeck process with an optimal rate of convergence. In this talk, we will consider generalizations of this methodology to the Langevin diffusion and the Landau equation. For the Langevin diffusion we will present Villani's entropic hypocoercivity theorem and our improvement. For the Landau equation we will present a linear functional inequality relating the entropy and the entropy production functional.

张 登 (上海交通大学)

Title: The three dimensional stochastic Zakharov system

Abstract: This talks concerns the three dimensional stochastic Zakharov system in the energy space, where the Schroedinger equation is driven by linear multiplicative noise and the wave equation is driven by additive noise. We will show the well-posedness of the system below the ground state. Furthermore, we will also present a noise regularization result which states that finite time blowup before any given time can be prevented with high probability by adding sufficiently large non-conservative noise.

张希承（北京理工大学）

Title: SDEs with supercritical distributional drifts

Abstract: Let $d \geq 2$. In this paper, we investigate the following stochastic differential equation (SDE) in $\mathbb{R}^d$ driven by Brownian motion

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \sqrt{2}\mathrm{d}W_t,$$

where b belongs to the space $\mathbb{L}_T^q \mathbf{H}_p^\alpha$ with $\alpha \in [-1,0]$ and $p,q \in [2,\infty)$, which is a distribution-valued and divergence-free vector field. In the subcritical case $\frac{d}{p} + \frac{2}{q} < 1 + \alpha$, we establish the existence and uniqueness of a weak solution to the integral equation:

$$X_t = X_0 + \lim_{n \to \infty} \int_0^t b_n(s, X_s)\mathrm{d}s + \sqrt{2}W_t.$$

Here, $b_n := b * \phi_n$ represents the mollifying approximation, and the limit is taken in the L^2-sense. In the supercritical case $1 + \alpha \leq \frac{d}{p} + \frac{2}{q} < 2 + \alpha$, if the initial distribution has an L^2-density, we show the existence of weak solutions as well as the associated Markov processes. Furthermore, if it is additionally assumed that $b = b_1 + b_2 + \operatorname{div} a$, where $b_1 \in \mathbb{L}_T^\infty \mathbf{B}_{\infty,2}^{-1}$, $b_2 \in \mathbb{L}_T^2 L^2$ and a is a bounded antisymmetric matrix-valued function, we also establish the convergence of mollifying approximation solutions without the need to subtract a subsequence.

郑术蓉（东北师范大学）

Title: Large separable sample covariance matrices: joint CLT for linear spectral statistics and its applications

Abstract: This paper studies a group of correlated separable sample covariance matrices of the form

$$\{n^{-1}\Gamma_n x_n B_{nt} B_{nt}^T x_n^T \Gamma_n^T, \, t = 1,\ldots, R\},$$

which share a latent random matrix x_n but have distinct spatial-temple covariance structures $\{\Gamma_n, B_{nt}\}$. The entries of the random matrix can be either independent and identically distributed or elliptically correlated across rows. A joint central limit theorem for linear spectral statistics of such covariance matrices is established in high-dimensional frameworks.

By utilizing this general result, we extend two classical likelihood ratio tests to high-dimensional situations, including the significance test in a multivariate linear regression and the test for the equality of several covariance matrices.

朱蓉禅 (北京理工大学)

Title: Large N limit and $1/N$ expansion of the observables for $O(N)$ linear sigma model

Abstract: In this talk, we talk about the large N problems for the Wick renormalized linear sigma model, i.e. N-component Φ^4 model, in two spatial dimensions, using stochastic quantization methods and Dyson--Schwinger equations. We identify the large N limiting law of a collection of Wick renormalized $O(N)$ invariant observables. In particular, under a suitable scaling, the quadratic observables converge in the large N limit to a mean-zero (singular) Gaussian field $\mathcal{Q}$ with an explicit covariance; and the observables which are renormalized powers of order $2n$ converge in the large N limit to suitably renormalized n-th powers of $\mathcal{Q}$.

Furthermore, we derive the $1/N$ expansion for the k-point functions of the quadratic observables by employing a graph representation and carefully analyzing the order of each graph from Dyson-Schwinger equations. Finally, we obtain the next order stationary dynamics.