



华罗庚应用数学讲座

A Low-rank Augmented Lagrangian Method for Polyhedral-SDP and Moment-SOS Relaxations of Polynomial Optimization

报告人: Kim-Chuan Toh 教授, 新加坡国立大学

时 间: 2026年7月22日(周三) 10:00-11:00

地 点: 中国科学院数学与系统科学研究院 南楼613

Abstract: Polynomial optimization problems (POPs) admit geometric convex conic reformulations (Kim–Kojima–Toh, SIOPT 30, 2020), though they remain NP-hard. We show that several well-known relaxations can be unified under a polyhedral-SDP framework obtained by approximating the intractable cone with intersections of polyhedral cones and the PSD cone. While yielding tight lower bounds, these relaxations become costly as variable dimensions and number of constraints scale as $\Omega(n^{2\tau})$ with the relaxation order τ .

We introduce RiNNAL-POP, a low-rank augmented Lagrangian method (ALM) to solve these large-scale polyhedral-SDP relaxations.

For $\Omega(n^{2\tau})$ nonnegativity and consistency constraints, we design a projection scheme whose cost is linear in the number of variables. We further expose facial structure in the relaxation and restrict the matrix variable to affine subspaces tied to exposed faces of the PSD cone, eliminating many linear constraints and enabling efficient factorized ALM subproblems. Each ALM iteration also applies a single projected-gradient step on the original matrix variable to adapt rank and escape spurious local minima when needed. We also extend the framework to moment-SOS relaxations. Experiments on various benchmarks show that RiNNAL-POP is robust and efficient for large-scale polyhedral-SDP relaxations.

[Based on joint work with Di Hou and Tianyun Tang]

报告人简介: Kim-Chuan Toh is a Provost's Chair Professor in the Department of Mathematics at the National University of Singapore. He works extensively on convex programming, particularly large-scale matrix optimization problems such as semidefinite programming, and sparse optimization problems arising from machine learning and statistics. Currently he serves as a co-Editor for Mathematical Programming Series A, an Area Editor for Mathematical Programming Computation, and Associate Editor for several journals including SIAM J. on Optimization, Operations Research, and ACM Transactions on Mathematical Software. He received the INFORMS Optimization Society Farkas Prize in 2017, and the triennial Mathematical Optimization Society Beale-Orchard Hays Prize in 2018 and Paul Tseng Memorial Lectureship in Continuous Optimization in 2024. He is a Fellow of the Singapore National Academy of Science, and a Fellow of SIAM

